

COMPUTATION OF REGIONAL TRAVEL TIMES AND STATION CORRECTIONS FROM THREE-DIMENSIONAL VELOCITY MODELS

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Summary

We have investigated the performance of a variety of methods to compute travel times at regional distances for heterogeneous three-dimensional velocity models of the Earth's crust and upper mantle. Ray-shooting and finite-difference methods give good results for the type of 3-D models analyzed. For S waves, the differences between methods are on the order of 0.5 seconds, which is sufficient for the generation of model-predicted station correction surfaces. The differences in travel times are mostly caused by the diversity in internal model representation between the travel time codes. For the models analyzed, the influence of off-great-circle propagation is small, particularly when compared to the differences in travel times between methods. Therefore, computations can be carried out in 2-D. For the purpose of calculating model-predicted stations corrections, the reference model of the 3-D model should be as close as possible to the 1-D reference model of the correction surface to avoid significant artifacts.

1. INTRODUCTION

The objective of this research is to evaluate methods for computing travel times at regional distances from three dimensional velocity models of the crust and uppermost mantle, and assess their usefulness in improving regional event locations

Locating low magnitude events with the accuracy required by the Comprehensive Nuclear-Test-Ban Treaty (CTBT) is a challenging task. These events are normally recorded only at regional distances, where the effect of crustal and upper mantle structure on seismic travel times is important. One-dimensional (1-D), radially-symmetric models such as iasp91

(Kennett and Engdahl, 1991) or ak135 (Kennett et al., 1995) cannot predict travel times accurately for local and regional distances. A method to account for the effects of lateral heterogeneity at a given station is to construct a source-specific station correction (SSSC). Empirical SSSCs can be constructed in regions where calibration data such as explosions and/or well located earthquakes are available (e.g., Bondár et al., 1999). Unfortunately large regions of the Earth have little or no calibration data and SSSCs must be obtained using available regional and/or global velocity models. To obtain these model-predicted SSSCs, rays have to be traced through the model, from each station to points on a specified latitude-longitude grid, within a given distance from the station (normally 20 degrees). Therefore, accurate and efficient methods to calculate travel times in 3-D velocity models are required.

2. TRAVEL TIME METHODS

Computation of travel times in heterogeneous media is a complicated problem. Many algorithms and computer codes have been developed, and each one has its advantages and shortcomings (for a recent review see Červený, 2001, chapter 3). The choice of a particular method or methods is then determined by factors such as the characteristics of the 3-D model, the required accuracy and computational efficiency, and many others. The 3-D model that we use in this study is a global shear velocity model of the crust and upper mantle obtained by inversion of broad-band surface wave velocities (Shapiro and Ritzwoller, 2001; hereafter referred to as SR2001). This model is parameterized in terms of vertical velocity profiles on each node of a 2×2 degree grid. For each vertical profile the crystalline crust consists of three layers, and there are two layers of sediments (in some regions one of both of the sedimentary layers might not be present). Below the Moho the velocity structure in the upper mantle is parameterized using splines. The model is transversely isotropic between the Moho and 220 km, but for the tests presented here only the equivalent isotropic model has been used. As a result of the parameterization the model might contain in many regions very thin layers and sharp velocity contrasts in depth. On the other hand, lateral velocity variations can also be strong, but are much smoother than in the vertical direction.

We have tested the suitability of three methods to compute rays and travel times through the SR2001 model: a ray-bending method based on perturbation theory, a ray-shooting method, and a finite-difference solver of the eikonal equation.

The ray bending code tested in this study is described by *Bijwaard and Spakman (1999)*. It is based on ray perturbation theory, and was developed by *Pulliam and Snieder (1996)* based on the original implementation of *Snieder and Sambridge (1993)*. This code performs 3-D ray tracing and computes accurate travel times very efficiently for models with small velocity perturbations relative to a spherically symmetric reference model (e.g. *Bijwaard et al., 1998*). However we failed to implement this method with the SR2001 model. The constant-velocity block parameterization used by the travel time code makes it difficult to accurately represent the SR2001 model. The model also contains very sharp velocity variations (particularly in depth) which violate the assumptions of the method, and produces unstable or inaccurate results for specific ray-paths. In these cases the reference ray can be very different from the minimum-time ray, and the method fails to converge rapidly and in some cases does not converge at all to the global minimum. Therefore this code could not be used in the comparison results presented in this study.

Ray-shooting methods use initial-value ray tracing to solve a two-point ray-tracing problem (*Červený, 2001, pp. 220-223*). This is accomplished by looping over different take-off angles and elementary waves until the ray that connects the source with a given receiver is found. Therefore this method is more time consuming than the bending method, but usually produces better results, particularly in presence of large velocity variations. The loop must be carefully designed because the ray field might be very complicated, and it is not uncommon the existence of regions (shadow zones) where no elementary wave arrives. On the other hand, the method naturally provides travel times for different wave types, not only first arrivals. As most ray-shooting methods, the one we have tested here (*Červený and Pšenčík, 1984*) has been implemented in 2-D, and does not consider off-great-circle propagation.

Finite difference methods are usually the fastest methods to compute travel times in heterogeneous media, although many implementations are not sufficiently accurate in regions with high velocity contrasts. In this study we have used the method of *Podvin and Lecomte (1991)* which combines computational efficiency with good performance even in regions containing extreme velocity variations of arbitrary shape. An important drawback of finite difference methods is that they usually only provide first-arrival travel times, and they do not produce rays. If these are needed, they can be obtained a posteriori by back-tracing from any point inside the model (receiver) to the source.

3. COMPARISON OF TRAVEL TIME METHODS

The comparisons presented here have been done for the SR2001 model, using the ray-shooting code of *Červený and Pšenčík (1984)* and the finite difference method of *Podvin and Lecomte (1991)*.

The ray shooting code of *Červený and Pšenčík (1984)* is a 2-D method, for laterally-varying layered models. The geometry of the layers is described in terms of splines, and the parameterization had to be refined in order to be able to include very thin, non-intersecting layers. Inside each layer the seismic velocity is parameterized in terms of smooth polynomials in both directions. An Earth-flattening transformation is applied to the model, and computations are then made in cartesian coordinates.

The finite difference technique of *Podvin and Lecomte (1991)* computes travel times in 3-D, and 2-D computations can also be done as a particular case. The velocity model is parameterized in terms of cubic constant-velocity cells in a cartesian reference frame. Therefore, models described using a spherical parameterization have to be mapped into a cartesian reference system. For 2-D computations this can be done applying an Earth-flattening transformation. For 3-D computations the spherical Earth has to be embedded inside a cube, resulting in some inefficiency in the amount of memory required to store the model.

Accuracy tests

We have conducted tests to evaluate the performance and accuracy of the ray shooting and finite difference methods. First we tested these methods against theoretical travel times for the one-dimensional, radially-symmetric model ak135 (*Kennett et al., 1995*). Second we compared the results of both methods in 2-D, using vertical cross-sections through the SR2001 model and finally we compare the performance of 2-D versus full 3-D travel times.

To evaluate the accuracy of the ray shooting and finite difference methods, we compare their results with theoretical travel times using the algorithm of *Buland and Chapman, (1983)* for ak135. The results of these comparisons are shown in Figure 1. In this case the ray shooting method is very precise, and the travel times are virtually identical to those obtained using

Buland and Chapman, (1983). The accuracy of the finite difference method depends on the cell size. For P waves, and a cell size of 1×1 km the precision is better than 0.05 s. If the cell size is increased to 2×2 km the precision is still better than 0.1 seconds, but some artifacts caused by the finite difference model parameterization (2×2 km constant velocity cells) can be seen. When the first arrival changes from Pg to Pn (at approximately 1.5 degrees) there is a baseline shift (Figure 1), which is caused because Moho depth in the finite difference model parameterization of 2×2 km cells is 36 km, instead of 35 km as in ak135.

Ray shooting versus finite differences

We have compared the 2-D ray shooting and the finite difference methods using great-circle vertical cross sections in Central Eurasia through the SR2001 model. For this comparison we have considered 2-D vertical profiles starting at various IMS stations in Central Eurasia, with azimuths ranging from 0 to 360 degrees. For all these profiles we calculate the RMS of the difference between the travel times computed using both methods as a function of distance. The results for the auxiliary IMS station AAK (Ala-Archa, Kyrgyzstan) using S waves are shown in Figure 2. The RMS of the difference increases between 0 and 200 km, and then stays rather constant, below 0.5 seconds, to distances of 2000 km. These differences of 0.4-0.5 seconds at regional distances and for S waves is much larger than the tests performed with the 1-D model ak135. We find that these differences are caused by the way the 3-D model is translated into the internal parameterization of each travel time code. In some other cases, not shown here, the differences are much larger, and caused by the failure of the ray-shooting method to find the minimum travel time ray path. In most cases, modifying the parameters that control the accuracy of the ray shooting solution eliminates these problems, but at the expense of longer computation time.

Importance of off-great-circle propagation

Finally we have investigated the importance of full 3-D travel time computation versus computations in 2-D. For this test we only use the finite-difference method of *Podvin and Lecomte (1991)* because the ray shooting code is only 2-D. To evaluate the differences between 3-D and 2-D travel times we have also considered various profiles through the SR2001 model. For all the profiles that we have tried, the differences between the computed 3-D and 2-D travel times are small (less than 0.1 seconds), and similar in magnitude to the

differences of the comparison with ak135 shown in Figure 1. Full 3-D ray-tracing is important for non-linear tomography studies, allowing to obtain improved, more focused velocity models (*Bijwaard and Spakman, 2000; Widiyantoro et al., 2000*), but for the model considered here and for regional distances it is not necessary for computing travel times and SSSCs with sufficient precision. This is probably because the model we have used is more heterogeneous vertically (with thin layers and sharp velocity contrasts) than laterally, and therefore off-great-circle propagation is not very important.

4. EFFECTS OF THE REFERENCE MODEL ON THE COMPUTATION OF SSSCS

Global velocity models obtained by inversion of large datasets of arrival times of seismic phases (e.g., *Bijwaard et al., 1998*), are normally described as perturbations to 1-D models such as iasp91 or ak135. These 3-D models provide high resolution in regions with high density of earthquakes and stations but contain large gaps in regions where seismicity is low and/or stations do not exist. On the other hand, global 3-D models that use surface wave and normal mode data have lower resolution than models based on body waves, but their coverage is more homogeneous. These models are normally described as perturbations relative to PREM (*Dziewonski and Anderson, 1981*)

There are some significant differences between PREM and ak135 in the crust and upper mantle. PREM is designed as an average Earth model, and contains a 3 km thick water layer, and the Moho discontinuity is located at 24.4 km depth. The crust in ak135 is of continental type (because most of seismic stations are located on continents), with a Moho depth of 35 km. In the uppermost mantle, between Moho and 220 km, PREM is transversely isotropic, and exhibits a well-developed low velocity zone, while ak135 is isotropic and lacks a low velocity zone, although the velocity gradient in the uppermost mantle is very small for both P and S. In the upper mantle PREM has a velocity discontinuity at 220 km depth which is much sharper for P than for S. This discontinuity is absent in ak135. Finally the transition zone is 20 km thicker for PREM (400 – 670 km) than for ak135 (410 – 660 km).

These differences between PREM and ak135 obviously translate into different travel times at regional distances. We have calculated the difference in travel times between PREM and ak135 for a source located at 40 km depth, and for receivers also at 40 km depth. This has been done to avoid additional complications by differences in crustal structure. Therefore,

the curves shown in Figure 3 represent the effect of differences in upper mantle and transition zone structure between the two models. Immediately below Moho v_{PH} for PREM is larger than v_P for ak135. As a result PREM travel times for v_{PH} are faster (smaller) than ak135, and this can be observed as increasingly negative values in Figure 3 (top panel, solid line). The difference in travel time reaches a minimum of approximately -3 s at distances of 14 degrees, and then decreases to 0 at 25 degrees. The situation is reversed for PREM v_{PV} . Immediately below Moho v_{PV} for PREM is slightly smaller v_P for ak135 and this results in positive values, indicating that travel times for PREM v_{PV} are slower (larger) than ak135 (top panel, dashed line). The same pattern can be observed for S wave travel times (Figure 3, bottom). In this case the differences in travel times are larger (-8 s for PREM v_{SH}) and the maximum values of the differences occur at different distances (i.e., at 19 degrees for PREM v_{SH}).

SSSCs are normally referred to iasp91 or ak135 (for the purpose of regional travel times both models are identical) but many global and regional models are expressed as perturbations relative to PREM. Because of the differences in travel times between PREM and ak135 we must expect artifacts in the SSSCs that are caused by the choice of the reference model, in addition to features due to lateral heterogeneity. Figure 4a shows the SSSC relative to ak135 for auxiliary IMS station AAK computed from the S model of *Villaseñor et al. (2001)*. At epicentral distances less than about 10 degrees, the character of the correction surface is dominated by the difference in crustal thickness between the 3-D model and ak135. The crust in Central Eurasia is thicker than 35 km, and this results in positive station corrections (slower travel times than ak135). Beyond 5-10 degrees, velocity variations in the uppermost mantle become increasingly important. High velocities in the Indian craton produce large negative corrections (faster travel times than ak135). This effect can also be observed in the East European platform, north of the Caspian Sea. On top of these features caused by lateral heterogeneity there is a ring of negative values, centered on the station and with a radius of approximately 20 degrees. As shown in Figure 3, this ring can be explained by the differences between the upper mantle 3-D model (which uses PREM as reference model) and ak135.

To avoid these artifacts, the SSSCs can be calculated relative to the model beneath the station, instead of relative to iasp91 or ak135. This guarantees that the velocity discontinuities of the 3-D model and reference model are located exactly at the same depths. Figure 4b shows the results of applying this procedure to AAK. The ring with negative

values has disappeared, the shape of the correction surface is smoother and better correlated with the features of the 3-D model. Figure 4c shows the difference between the two correction surfaces obtained for AAK (Figures 4a and 4b). This clearly illustrates that the ring is an artifact caused by the choice of the reference model.

5. CONCLUSIONS

We have tested the performance of computer codes based on ray bending, ray shooting and finite differences for computing travel times and model-predicted SSSCs. For three-dimensional Earth models with large lateral and vertical velocity variations ray shooting and finite differences provide the best results.

For laterally heterogeneous models, the differences in computed travel times between different codes can be up to 0.5 seconds for regional distances. In some cases this can be caused by poor performance of one the methods (for example not being able to find the minimum travel time path, and converging to a local minimum). However, in the examples presented here, the differences are caused by differences in the internal model parameterization of each travel time code. These differences can be larger if the influence of model uncertainties on the computation of travel times is considered.

Full 3-D ray tracing is important for non-linear tomography studies, allowing to obtain improved, more focused velocity models. However, 3-D travel times are not necessary for computing regional SSSCs with models like the ones used in this study. Travel time differences between 2-D and 3-D ray tracing are much smaller in magnitude to differences between travel time codes. In the future, when higher-resolution, more heterogeneous models become available, 3-D ray tracing may be required.

The choice of the reference model is extremely important for determining the features of model-predicted SSSCs. To ensure smoothly varying SSSCs, the 3-D model should have the upper mantle and transition zone discontinuities at the same depth as the reference model. Otherwise, the computed SSSCs may display large artifacts, that manifest as circular features (positive or negative in sign) centered on the station. These artifacts can be eliminated by using the model beneath the station as the reference model for the computation of the SSSCs.

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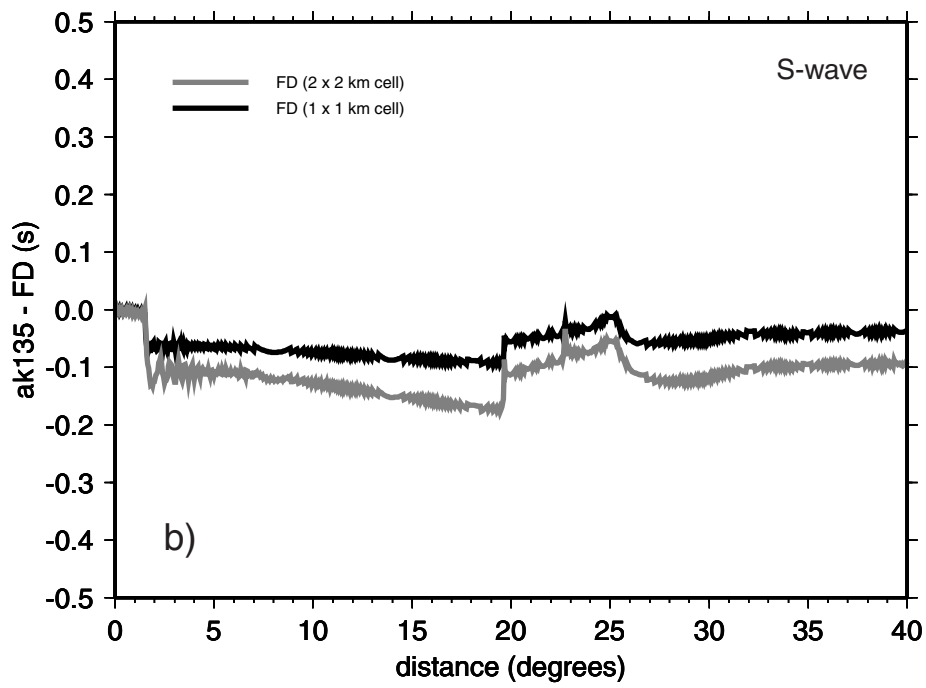
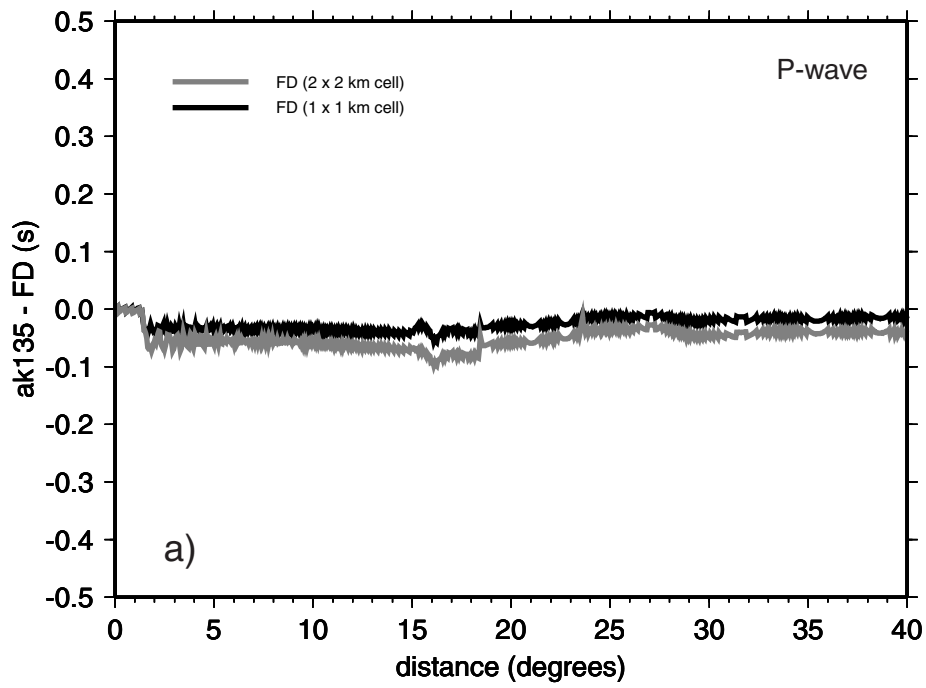
FIGURE CAPTIONS

Figure 1. Differences between computed travel times with finite differences varying the cell size, and theoretical arrival times for the one dimensional model ak135. Thick black line for the 1×1 km cells, thick gray line for 2×2 km cells: a) results for P waves, and b) for S waves.

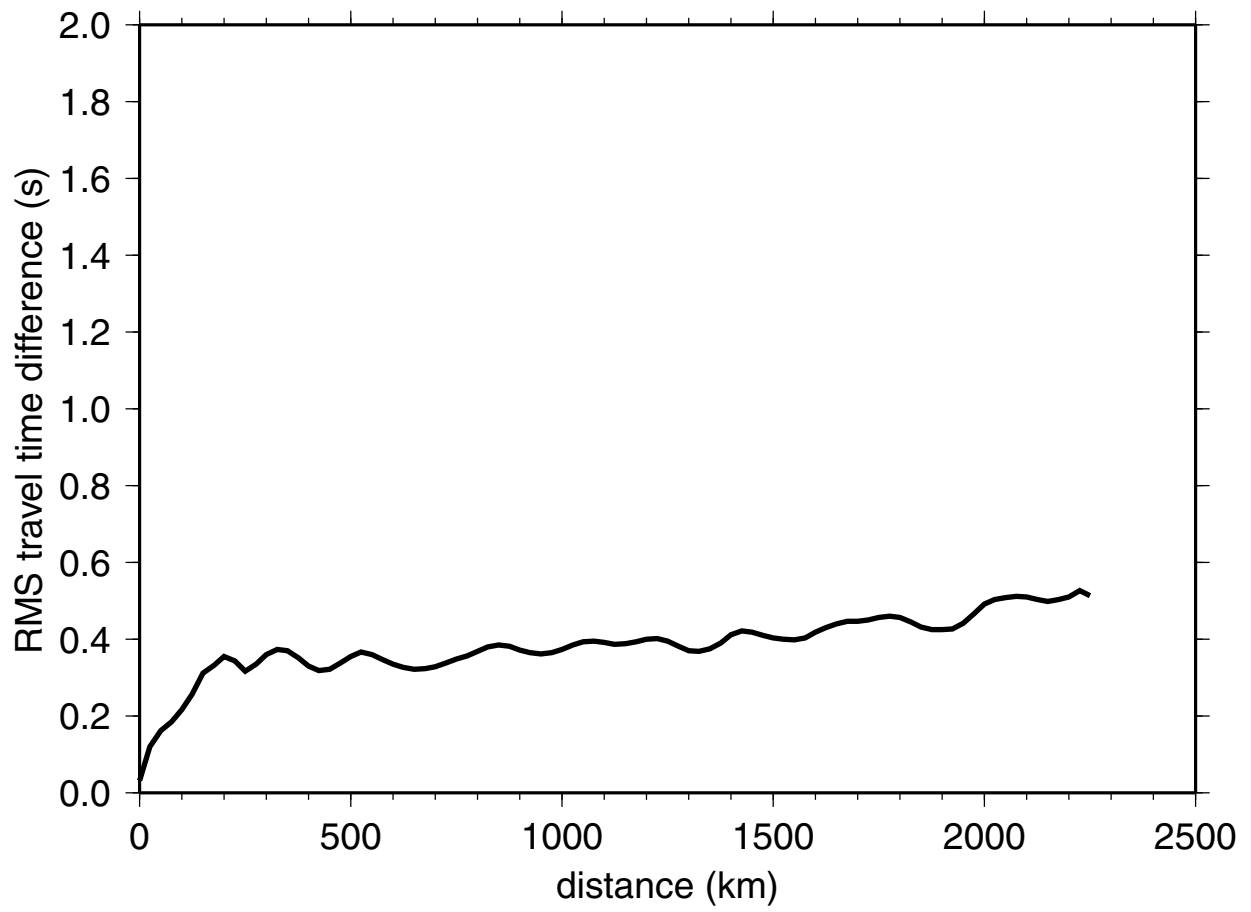
Figure 2. RMS difference as a function of distance between travel times computed using the 2-D ray shooting and the 2-D finite difference methods through the S model of SR2001 (*Shapiro and Ritzwoller, 2001*). The 2-D profiles used here start at IMS station AAK (Ala-Archa, Kyrgyzstan) and have azimuths from 0 to 360 degrees.

Figure 3. Travel time differences between 1-D models PREM and ak135 for regional distances. Top panel shows differences in P-wave traveltimes, and bottom panel shows differences in S-wave travel times.

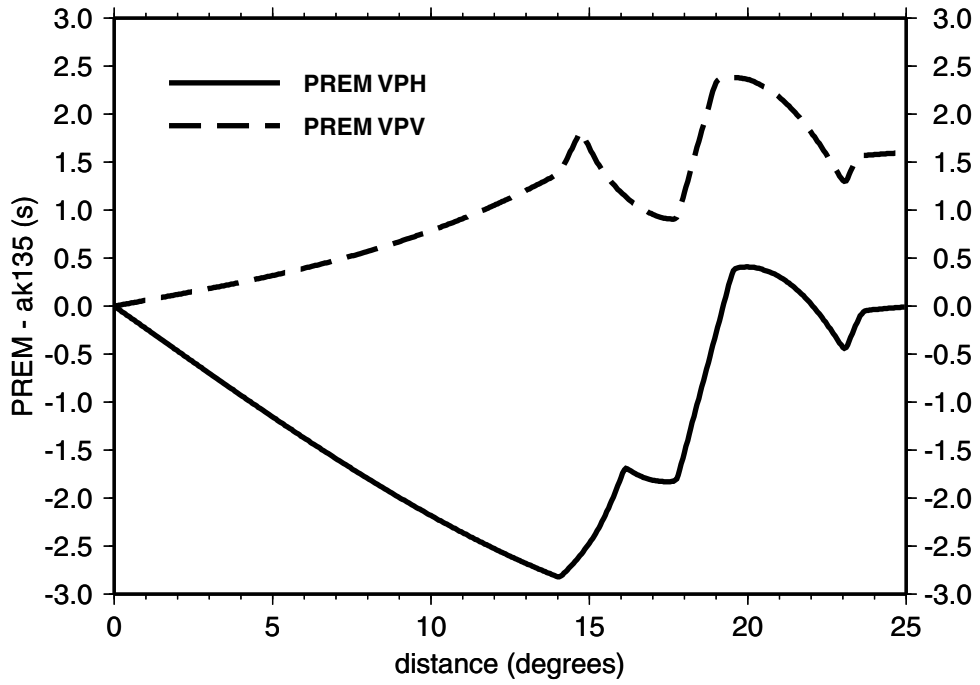
Figure 4. SSSCs for IMS station AAK (Ala-Archa, Kyrgyzstan) for the S model of SR2001 (*Shapiro and Ritzwoller, 2001*): a) correction surface relative to ak135, b) relative to the model beneath the station, and c) difference between the two SSSCs.



S travel times (FD - RS)



Difference in P wave travel times



Difference in S wave travel times

